

HONG KONG DIPLOMA OF SECONDARY
EDUCATION EXAMINATION MOCK PAPER

MATHEMATICS M1
**PAPER: CALCULUS AND STATISTICS (2027
ONWARDS)**

Time Allowed: 2 hr 30 min

Total Marks: 100

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of multiple sections.
2. Answer ALL questions in this paper.
3. Write your answers in the spaces provided.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

SECTION A

QUESTION 1

Answer all parts.

- (a) Find the coefficient of x^3 in the expansion of $(1 + 2x)^5$. (2 marks)

- (b) Find the first three terms, in ascending powers of x , in the expansion of $(1 - 3x)^{-2}$. (3 marks)

QUESTION 2

Answer all parts.

- (a) Solve $e^{2x} = 7$ for x , giving your answer to 3 significant figures. (2 marks)

- (b) Solve $\log_2(x - 1) + \log_2(x + 1) = 3$, stating any restriction on x . (3 marks)

QUESTION 3

Let $f(x) = x^3 - 4x^2 + 5x + 2$.

- (a) Find $f'(x)$. (2 marks)

Question 3 (continued)

- (b) Find $f''(x)$ and hence find the value of $f''(2)$. (3 marks)

QUESTION 4

The height h metres of a particle above a horizontal surface after t seconds is $h(t) = 12 + 8t - 2t^2$, for $0 \leq t \leq 4$.

- (a) Find the time at which the particle reaches its maximum height. (3 marks)

- (b) Find this maximum height. (2 marks)

QUESTION 5

Answer all parts.

- (a) Find $\int (6x^2 - 4x + 3) dx$. (3 marks)

- (b) A particle has velocity $v(t) = 6t - 4 \text{ m s}^{-1}$. Given that its displacement from a fixed origin is 5 m when $t = 0$, (2 marks)
find its displacement at time t .

QUESTION 6

The curve $y = 2x + 1$ is considered for $0 \leq x \leq 4$.

- (a) Find the exact area under the curve and above the x -axis from $x = 0$ to $x = 4$. (2 marks)

Question 6 (continued)

- (b) Estimate the same area using the trapezoidal rule with 4 equal strips. (3 marks)

QUESTION 7

A factory uses machines A and B to produce 60% and 40% of its components respectively. The probability that a component from A is defective is 0.02, while that from B is 0.05.

- (a) Find the probability that a randomly selected component is defective. (2 marks)
- (b) Given that a selected component is defective, find the probability that it was produced by machine B . (3 marks)

QUESTION 8

A discrete random variable X has probability distribution $P(X = x) = kx$ for $x = 1, 2, 3$.

- (a) Find k . (2 marks)
- (b) Find $E(X)$. (2 marks)
- (c) Find $\text{Var}(X)$. (1 mark)

QUESTION 9

Answer all parts.

- (a) A fair coin is tossed 5 times. Find the probability of obtaining exactly 3 heads. (3 marks)
- (b) The number of calls received by a switchboard in a minute follows a Poisson distribution with mean 2. Find the probability of receiving no calls in a particular minute. (2 marks)

QUESTION 10

The random variable X is normally distributed with mean 50 and standard deviation 8.

Standard normal distribution values

z	$P(Z < z)$
-1.00	0.1587
1.00	0.8413
1.50	0.9332

- (a) Find $P(X > 62)$. (3 marks)
- (b) Find c such that $P(X < c) = 0.1587$. (2 marks)

SECTION B

QUESTION 11

A rectangular garden is to be built beside a straight wall, so fencing is required for only three sides. The total length of fencing available is 24 m. Let x m be the width perpendicular to the wall and y m be the length parallel to the wall.

(a) Express y in terms of x . (2 marks)

(b) Find the value of x that gives the maximum area of the garden. (4 marks)

(c) Find the maximum area. (2 marks)

(d) For this value of x , find the area of the garden by evaluating a definite integral of its width profile (4 marks)

$w(u) = x - \frac{u}{6}$ for $0 \leq u \leq 6$. Give the value in square metres.

QUESTION 12

A quality-control test for a component has probability 0.9 of giving a positive result when the component is defective, and probability 0.08 of giving a positive result when it is not defective. A production line makes defective components with probability 0.04. Components are tested independently. Let X be the number of defective components among 20 randomly selected components.

(a) Find the probability that a randomly selected component gives a positive test result. (4 marks)

(b) Given that a component gives a positive result, find the probability that it is defective. (4 marks)

(c) State the distribution of X and find $P(X \geq 2)$. (4 marks)

QUESTION 13

The lifetimes, in hours, of a certain type of battery are normally distributed with mean μ hours and known standard deviation 12 hours. A random sample of 64 batteries has sample mean lifetime 72 hours.

(a) State the sampling distribution of the sample mean $\bar{X}$ in terms of μ , and find its standard error. (3 marks)

(b) Using the sample, find a 95% confidence interval for μ . Use $z_{0.975} = 1.96$. (4 marks)

Question 13 (continued)

- (c) Find the probability that, for a sample of 64 batteries from this population, the sample mean exceeds 75 hours if (3 marks)

$$\mu = 72.$$

- (d) How large a sample is required so that the 95% confidence interval for μ has margin of error at most 2 hours? (3 marks)

$$\text{Use } z_{0.975} = 1.96.$$

QUESTION 14

A manufacturer models the number of minor faults found in a batch of products by a Poisson distribution with mean λ faults per batch. In a sample of 50 batches, the observed numbers of faults were recorded as follows.

Observed number of faults in 50 batches

Number of faults, x	0	1	2	3	4 or more
Number of batches	12	18	12	6	2

- (a) Use the midpoint values 0, 1, 2, 3, 4 for the respective categories to estimate λ from the sample. (4 marks)

- (b) Using your estimate of λ , find the probability that a future batch has no faults. (4 marks)

Question 14 (continued)

- (c) Using your estimate of λ , find the probability that at least 3 faults occur in a combined production run of 2 (5 marks)
independent batches. State the distribution used and give your answer to 4 decimal places.