

HONG KONG DIPLOMA OF SECONDARY
EDUCATION EXAMINATION MOCK PAPER

MATHEMATICS M2
PAPER: ALGEBRA AND CALCULUS (2027
ONWARDS)

Time Allowed: 2 hr 30 min

Total Marks: 100

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of multiple sections.
2. Answer ALL questions in this paper.
3. Write your answers in the spaces provided.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

A

QUESTION 1

Let $f(x) = \frac{x^3}{x^2 + 1} + \cos x$.

(a) Determine whether $g(x) = \frac{x^3}{x^2 + 1}$ is odd, even or neither. Give a reason. (2 marks)

(b) Determine whether $h(x) = \cos x$ is odd, even or neither. (1 mark)

(c) Find $f(2) + f(-2)$. (2 marks)

QUESTION 2

Use mathematical induction to prove that, for every positive integer n , $1 + 3 + 5 + \cdots + (2n - 1) = n^2$. (5 marks)

QUESTION 3

Find the coefficient of x^3 in the expansion of $(1 + 2x)^5$.

(5 marks)

QUESTION 4

You may use $\sin 2x = 2 \sin x \cos x$ and $\cos x = \sin\left(\frac{\pi}{2} - x\right)$. Solve $\sin 2x = \cos x$ for $0 \leq x < 2\pi$. Give your answers in radians.

(5 marks)

QUESTION 5

Using $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$, evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{2x}$.

(5 marks)

QUESTION 6

The curve C has equation $y = (x^2 + 1)e^x$.

(a) Find $\frac{dy}{dx}$. (3 marks)

(b) Find the gradient of C at $x = 0$. (2 marks)

QUESTION 7

A function G satisfies $G'(x) = 3x^2 - 4x + 2$ and $G(0) = 1$.

(a) Find $G(x)$. (3 marks)

(b) Hence find $G(2)$. (2 marks)

QUESTION 8

Find the area enclosed by the curve $y = x^2 - 1$ and the x -axis between $x = -1$ and $x = 2$. (5 marks)

QUESTION 9

Consider the simultaneous equations $2x + y = 5$ and $x - y = 1$.

(a) Find the determinant of the coefficient matrix. (2 marks)

(b) Solve the simultaneous equations. (3 marks)

QUESTION 10

Let $\mathbf{a} = (1, 2, 0)$ and $\mathbf{b} = (2, -1, 2)$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$. (2 marks)

(b) Find $\mathbf{a} \times \mathbf{b}$. (3 marks)

B

QUESTION 11

Consider $f(x) = xe^{-x}$ for $x \geq 0$.

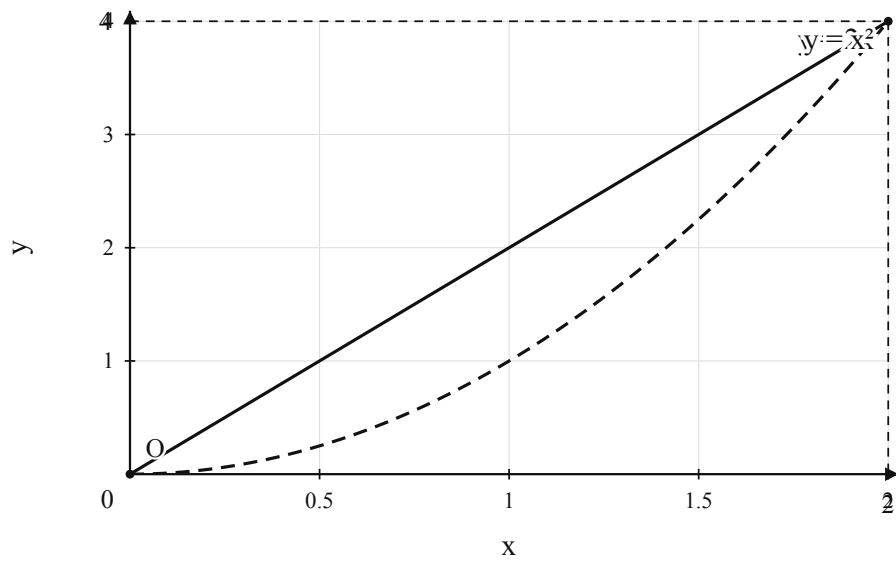
(a) Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$. (3 marks)

(b) Find the stationary point of f and determine its nature. (5 marks)

(c) Find the equation of the tangent to the graph of $y = f(x)$ at $x = 2$. (4 marks)

QUESTION 12

Consider the curves $y = 2x$ and $y = x^2$ for $0 \leq x \leq 2$.



- (a) Find $\int (2x - x^2) dx$. (4 marks)
- (b) Find the area enclosed by the two curves. (5 marks)
- (c) Find the value a in $0 < a < 2$ such that the area between the curves from $x = 0$ to $x = a$ is half the enclosed area. (4 marks)

QUESTION 13

For a real parameter λ , consider the system $\lambda x + y = 1$, $x + \lambda y = 1$, whose coefficient matrix is $A = \begin{pmatrix} \lambda & 1 \\ 1 & \lambda \end{pmatrix}$.

(a) Find $\det A$. (2 marks)

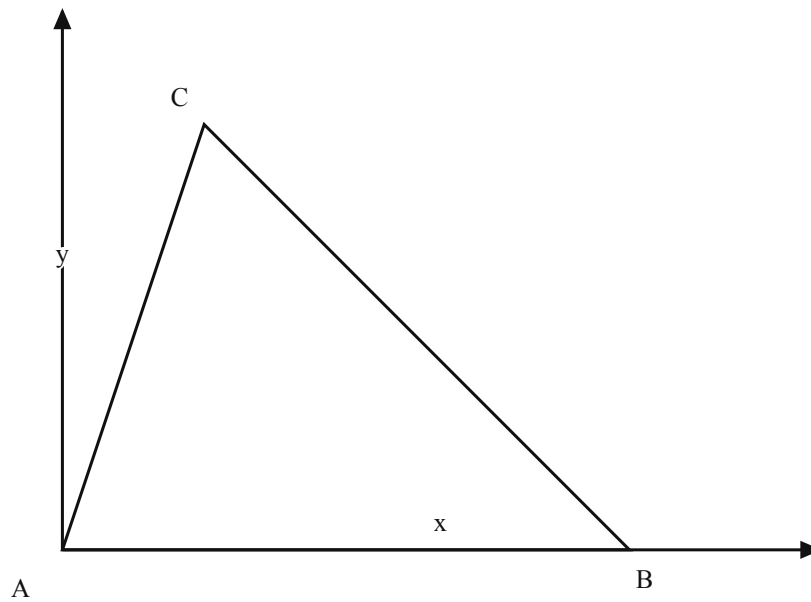
(b) State the values of λ for which the system has a unique solution. (2 marks)

(c) Solve the system when $\lambda = 2$. (4 marks)

(d) Discuss the system when $\lambda = 1$ and when $\lambda = -1$. (4 marks)

QUESTION 14

In three-dimensional space, let $A = (0, 0, 0)$, $B = (4, 0, 0)$ and $C = (1, 3, 0)$. The diagram shows the triangle ABC in the xy -plane.



- (a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (2 marks)
- (b) Find the angle BAC , giving your answer to the nearest degree. (4 marks)
- (c) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence find the area of triangle ABC . (4 marks)

Question 14 (continued)

- (d) Find a unit vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(3 marks)